

# Finsler metrics and CPT

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The role of Finsler-like metrics in situations where Lorentz symmetry breaking and also CPT violation are discussed. Various physical instances of such metrics both in quantum gravity and analogue systems are discussed. Both differences and similarities between the cases will be emphasised. In particular the medium of D-particles that arise in string theory will be examined. In this case the breaking of Lorentz invariance, at the level of quantum fluctuations, together with concomitant CPT in certain situations will be analysed. In particular it will be shown correlations for neutral meson pairs will be modified and a new contribution to baryogenesis will appear.

## I. INTRODUCTION

Paul Finsler in 1918 wrote a thesis, “Über Kurven und Flächen in Allgemeiner Rämen”, which is a generalisation of Riemannian manifolds. It remained a topic in the province of pure mathematics until quite recently. However, within physics, it has been realised more recently that a framework more general than pseudo-Riemannian geometry is required if causal relations, which are more complicated than special relativity, are involved. It is interesting that this realisation has sprung up in somewhat diverse contexts. We shall be primarily interested in the particular context known as space-time D foam, where it appears in a more generalised way. In general, the mathematical structure that is unveiled in physical applications, is different from the original Finsler theory in that the associated norm is not positive definite; hence, following physics tradition, it can be dubbed pseudo-Finsler. However, in the foamy context, a stochastic aspect to the norm is introduced.

At each point  $x$  in a (standard) Finsler manifold  $M$  [1] there is a norm on the tangent space  $TM_x$  which has not been induced by an inner product. Rather the norm itself induces an inner product. However this inner product is not parametrised by points in  $M$  but by directions in  $TM$ . Specifically a Finsler norm  $F$  on  $TM$  is a smooth mapping on  $TM \setminus \{0\}$  ( $\equiv \cup \{T_x M \setminus \{0\} : x \in M\}$ ). Now  $F|_{T_x M} : T_x M \rightarrow [0, \infty)$  is such that  $F|_{T_x M} (\equiv f_x)$  is homogeneous of degree 1 and, for all  $y$  in  $TM \setminus \{0\}$ , a form  $g_y : T_x M \times T_x M \rightarrow \mathbb{R}$  can be defined by

$$g_y : (u, v) \rightarrow \frac{1}{2} \frac{\partial^2 \left\{ [f_x(y + pu + qv)]^2 \right\}}{\partial p \partial q} \Big|_{p=q=0}.$$

The form  $g_y$  is bilinear and required to be positive definite. A Riemannian manifold  $(M, G)$ ,  $G$  being the metric, trivially can be recast as a Finsler manifold with

$$F(x, y) = \sqrt{G_x(y, y)}.$$

Here  $x$  is a point in  $M$ .

### A. The stochastic Finsler metric

String theory currently attracts much attention in that quantum states of gravitons are part of its spectrum together with other states which are required for the standard model of particle physics. The discovery of new solitonic structures in superstring theory [2] has dramatically changed the understanding of target space structure. These new non-perturbative objects are known as D-branes and their inclusion leads to a scattering picture of space-time fluctuations. Typically open strings interact with D-particles and satisfy Dirichlet boundary conditions when attached to them. Closed strings may be cut by D-particles. D-particles are allowed in certain string theories such as bosonic, type IIA and type I and here we will here consider them to be present in string theories of phenomenological interest. Furthermore even when elementary D particles cannot exist consistently there can be effective D-particles formed by the compactification of higher dimensional D branes. Moreover D particles are non-perturbative constructions since their masses are inversely proportional to the the string coupling  $g_s$ . The study of D-brane dynamics has been made possible by Polchinski's realization that such solitonic string backgrounds can be described in a conformally invariant way in terms of world sheets with boundaries. On these boundaries Dirichlet boundary conditions for the collective target-space coordinates of the soliton are imposed. Heuristically, when low energy matter given by a closed

(or open) string propagating in a  $(D + 1)$ -dimensional space-time collides with a very massive D-particle embedded in this space-time, the D-particle recoils as a result. Since there are no rigid bodies in general relativity the recoil fluctuations of the brane and their effectively stochastic back-reaction on space-time cannot be neglected. Based on these considerations, a model for a supersymmetric space-time foam has been suggested in [3]. The model is based on parallel brane worlds (with three spatial large dimensions), moving in a bulk space time which contains a “gas” of D-particles. The number of parallel branes used is dictated by the requirements of target-space supersymmetry in the limit of zero-velocity branes. One of these branes represents allegedly our Observable Universe. As the brane moves in the bulk space, D-particles cross the brane in a random way. From the point of view of an observer in the brane the crossing D-particles will appear as flashing on and off space-time defects, that is microscopic space-time fluctuations. This will give the four-dimensional brane world a “D-foamy” structure.

Interactions in string theory, at the present moment, are not treated systematically as a second quantised formalism is lacking. An important consistency requirement of first quantised string theory is conformal invariance, which determines the space-time dimension and/or structure. On the brane there are closed and open strings propagating. Each time these strings “meet” a D-particle, there is a possibility of being attached to it. The entangled state causes a back reaction onto the space-time, which can be calculated perturbatively using logarithmic conformal field theory formalism [4]. Some details are reviewed in Appendix A.

Let  $M_{QG} \left( = \frac{M_S}{g_s} \right)$  be the quantum gravity mass scale with  $g_s$  being the string coupling and  $M_S$  the string mass scale. Even at low energies  $E$ , decoherence effects of this foam can have magnitude  $O \left( \left[ \frac{E}{M_{QG}} \right]^n \right)$ , where  $n$  is model-dependent with values. Usually  $M_{QG}$  is taken to be  $M_P$ , the Planck mass. Hence, if  $M_{QG}$  can be decreased by many orders of magnitude, then the decoherence effect is duly enhanced. This decoherence is due to the topologically non-trivial interactions of the D-particles with strings. We shall see that in some models, involving large extra dimensions,  $M_{QG}$  can reduce to TeV scale. Such models were devised to deal with the hierarchy of scales between the Planck and electro-weak scales. Indeed, in the absence of large extra dimensions, at the heuristic level within the context of microscopic black holes, two quarks inside a proton can be absorbed by a virtual black hole with a re-emission into an antiquark and lepton. An estimate for the probability of this process can be given in terms of the quantum chromodynamics scale ( $\Lambda_{QCD}$ ) and  $M_{QG}$ . Typically the proton size is  $\Lambda_{QCD}^{-1}$ , and so, the probability of two quarks to come within a distance  $l_P$  within the lifetime of the black hole, is  $O \left( \left( \frac{\Lambda_{QCD}}{M_{QG}} \right)^4 \right)$ . The constraint on proton lifetime being  $10^{33}$ yr or longer implies that  $M_{QG} > 10^{16}$ GeV[5]. Fast baryon decay can be inhibited by separating the quark and lepton in a large extra dimension, resulting in overlaps of wavefunctions being reduced and a smaller  $M_{QG}$ . A model[6] which can produce  $M_{QG}$  of TeV size has been proposed and involves a  $4 + n$  dimensional Universe with structure  $M_4 \times K_n$ , where  $M_4$  is a  $3 + 1$  dimensional domain wall (brane) and  $K_n$  is a flat  $n$ -dimensional manifold with extra dimensions of size  $R$ . In the model, all standard model interactions in particle physics would be confined on  $M_4$ , but not gravity. This leads to an  $M_{QG}$  satisfying:

$$M_{QG} = \frac{1}{2\pi R} (2\pi R M_P)^{\frac{2}{n+2}}.$$

For example if  $n = 2$  and  $R = 0.1$ mm then  $M_{QG} = 10$ TeV.

Using the brane model [7] for space-time fluctuations one can obtain the following expression for the induced space-time distortion as a result of the D-particle recoil, in the weakly coupled string limit, which will be appropriately used in :

$$g_{ij} = \delta_{ij}, g_{00} = -1, g_{0i} = \varepsilon (\varepsilon y_i + u_i t) \Theta_\varepsilon(t), i = 1, \dots, D \quad (1.1)$$

where the suffix 0 denotes temporal (Liouville) components and

$$\Theta_\varepsilon(t) = \frac{1}{2\pi i} \int_{-\infty}^{\infty} \frac{dq}{q - i\varepsilon} e^{iqt}, \quad (1.2)$$

$$u_i = (k_1 - k_2)_i, \quad (1.3)$$

with  $k_1$  ( $k_2$ ) the momentum of the propagating closed-string state before (after) the recoil;  $y_i$  are the spatial collective coordinates of the D particle and  $\varepsilon^{-2}$  is identified with the target Minkowski time  $t$  for  $t \gg 0$  after the collision [4] (see Appendix A). These relations have been calculated for non-relativistic branes where  $u_i$  is small and require the machinery of logarithmic conformal field theory. Now for large  $t$ , to leading order,

$$g_{0i} \simeq \bar{u}_i \equiv \frac{u_i}{\varepsilon} \propto \frac{\Delta p_i}{M_P} \quad (1.4)$$

where  $\Delta p_i$  is the momentum transfer during a collision and  $M_P$  is the Planck mass (actually, to be more precise  $M_P = M_s/g_s$ , where  $g_s < 1$  is the (weak) string coupling, and  $M_s$  is a string mass scale); so  $g_{0i}$  is constant in space-time but depends on the energy content of the low energy particle.

### B. Decoherence and non-unitary evolution

The recoiling D-particle background leads to a fluctuating background. In string theory for fixed backgrounds to be consistent with a classical space-time interpretation the central charge needs to have the critical value. Quantum scattering off the D-particle excites an open string state which attaches itself to the D-particle. Closed-to-open string amplitudes determine these excitation and emission processes. To allow in principle for fluctuating backgrounds it is necessary to deform the conformal points through vertex operators  $V_{g^I}$  associated with background fields  $g^I$ . The world-sheet action  $S_\sigma$  has a structure

$$S_\sigma = S^* + g^I \int_\Sigma V_I(\Xi) d^2\xi,$$

which is a (weakly) non-conformal deformation of  $S^*$ , the conformal action,  $V_I$  is a vertex (logarithmic conformal field theory) operator associated with the D-particle,  $\Xi$  are target space matter fields,  $\Xi$  is the usual holomorphic world sheet co-ordinate, and  $\Sigma$  is the world-sheet surface. In our case the relevant  $g^I = u_i$  with  $i = 1, 2, 3$ , if the D-particle recoil is confined to the D3 brane on which matter strings propagate. Since we are interested in the dynamics of matter, we will consider the reduced density matrix of matter  $\rho_M$  on a fixed genus world-sheet.

The renormalization group for field theories in two dimensions has properties which will have an interesting reinterpretation for string theories [8, 9]. In theory space, i.e. in the infinite dimensional space of couplings  $\{g^I\}$ , for all renormalizable two-dimensional unitary quantum field theories, there exists a function  $c(\{g^I\})$  which has the following properties:

- $c(\{g^I\})$  is non-negative and non-increasing on renormalization group flows towards an infrared fixed point
- the renormalization group fixed points are also critical points of  $c(\{g^I\})$
- critical value of  $c(\{g^I\})$  is the conformal anomaly

More precisely in terms of a renormalization group flow parameter (scale)  $t$

$$\frac{d}{dt}c(\{g^I\}) = \beta^I G_{IJ} \beta^J \quad (1.5)$$

where the renormalization group  $\beta$  function is defined by

$$\beta^I = \frac{d}{dt}g^I(t)$$

and  $G_{IJ}$  is referred to as the Zamolodchikov metric.  $G_{IJ}$  is negative definite and is the matrix of second derivatives of the free energy. In the approach to perturbative string theory based on conformal field theory on the world sheet, it has long been thought that the time evolution of the string backgrounds and world sheet renormalization group flows are connected [10][11]. In non-critical strings conformal invariance is restored by gravitational dressing with a Liouville field  $\varphi$  (which can be viewed as a local world sheet scale) so that (for convenience now dropping the index  $I$ )

$$\int d^2z g V_g(\Xi) \rightarrow \int d^2z g e^{\alpha_g \varphi} V_g(\Xi). \quad (1.6)$$

However  $V_g$  has a scaling dimension  $\alpha_g$  to  $O(g)$  with  $\alpha_g = -g C_{ggg} + \dots$  and  $C_{ggg}$  is the expansion coefficient in the operator product expansion of  $V_g$  with itself. In a small  $g$  expansion

$$\int d^2z g V_g(\Xi) \rightarrow \int d^2z g V_g(\Xi) - \int d^2z g^2 C_{ggg} \varphi V_g(\Xi).$$

Scale invariance is restored by defining a renormalized coupling  $g_R$

$$g_R = g - C_{ggg} \varphi g^2 \quad (1.7)$$

The local scale interpretation of  $\varphi$  is clearly consistent with the renormalisation group  $\beta$  function that we have defined earlier. On noting that  $\gamma_{\alpha\beta} = e^{\varphi} \hat{\gamma}_{\alpha\beta}$  where  $\hat{\gamma}_{\alpha\beta}$  is a fiducial metric, the integration over world sheet metrics  $\gamma_{\alpha\beta}$  (in the Polyakov string action) implies an integration over  $\varphi$ . In this way  $\varphi$  becomes a dynamical variable with a kinetic term. For matter fields with central charge  $c_m > 25$  the signature of this term is opposite to the kinetic terms for the fields  $\Xi$  and it has been suggested that in this case the zero mode of  $\varphi$  is a target time  $t$ . The requirement of renormalizability of the world sheet  $\sigma$  model implies that for the density matrix  $\rho_M$  of a string state propagating in a background  $\{g_i\}$

$$\frac{d}{dt} \rho_M(g^I, p_I, t) = 0 \quad (1.8)$$

where  $p_i$  is the conjugate momentum to  $g^i$  within the framework of a dynamical system with hamiltonian  $H$  and action the Zamolodchikov c-function [12], i.e.

$$c[g] = \int dt (p_I \dot{g}^I - H). \quad (1.9)$$

From (\ref{density}) we deduce that

$$\frac{\partial \rho}{\partial t} + \dot{g}^I \frac{\partial \rho}{\partial g^I} + \dot{p}_I \frac{\partial \rho}{\partial p^I} = 0. \quad (1.10)$$

The piece  $\dot{p}_I \frac{\partial \rho}{\partial p^I}$  in (1.10) can be written as  $G_{IJ} \beta^J \frac{\partial \rho}{\partial p^I}$ ; using the canonical relationship of  $g^I$  and  $p^I$  this can be recast as  $-i G_{IJ} \beta^J [\rho, g^I]$ . As discussed in [12], such a term leads to a non-unitary evolution of  $\rho$ . This analysis cannot be regarded as being conclusive concerning the issue of non-unitarity, even within the framework of non-critical string theory, since there are issues relating to the time like signature of the Liouville field and the identification of the local renormalization group scale. However we will assume that these caveats are not sufficient to invalidate the main consequence and we shall explore the implications for  $CPT$  ( $\equiv \Theta$ ) symmetry where the operators  $C$ ,  $P$ , and  $T$  denote charge conjugation, parity and time reversal respectively.

### C. CPT

Currently the successful theories are based on local Lorentz invariant lagrangians, and it has been shown given the spin-statistics connection that  $CPT$  is a symmetry. The symmetry implies that the solution set of a theory is invariant under reversal of parity, time and interchange of particle and antiparticle. Consequently any violations of the consequences of  $CPT$  symmetry [13] would entail physics beyond the standard model of particle physics which is based on lagrangians. Typically the consequences of  $CPT$  that are considered are those of equal masses and lifetimes for particles and antiparticles. Recently it was noted that when the  $CPT$  operator is not well defined there are implications for the symmetry structure of the initial entangled state of two neutral mesons in meson factories such as DAΦNE, the Frascati  $\phi$  factory. Indeed, if  $CPT$  can be defined as a quantum mechanical operator, then the decay of a (generic) meson with quantum numbers  $J^{PC} = 1^{--}$  [14], leads to a pair state of neutral mesons  $|i\rangle$  having the form of the entangled state

$$|i\rangle = \frac{1}{\sqrt{2}} \left( |\overline{M}_0(\vec{k})\rangle |M_0(-\vec{k})\rangle - |M_0(\vec{k})\rangle |\overline{M}_0(-\vec{k})\rangle \right). \quad (1.11)$$

This state has the Bose symmetry associated with particle-antiparticle indistinguishability  $C\mathcal{P} = +$ , where  $C$  is the charge conjugation and  $\mathcal{P}$  is the permutation operation. If, however,  $CPT$  is not a good symmetry (i.e. ill-defined due to space-time foam), then  $M_0$  and  $\overline{M}_0$  may not be identified and the requirement of  $C\mathcal{P}$  is relaxed [15]. Consequently, in a perturbative framework, the state of the meson pair can be parametrised to have the following form:

$$|i\rangle = \frac{1}{\sqrt{2}} \left( |\overline{M}_0(\vec{k})\rangle |M_0(-\vec{k})\rangle - |M_0(\vec{k})\rangle |\overline{M}_0(-\vec{k})\rangle \right) + \frac{\omega}{\sqrt{2}} |\Delta(\vec{k})\rangle$$

where

$$|\Delta(\vec{k})\rangle \equiv |\overline{M}(\vec{k})\rangle |M_0(-\vec{k})\rangle + |M_0(\vec{k})\rangle |\overline{M}(-\vec{k})\rangle$$

and  $\omega = |\omega| e^{i\Omega}$  is a complex *CPT* violating (*CPTV*) parameter. For definiteness in what follows we shall term this quantum-gravity effect in the initial state [15].

Decoherence or non-unitary evolution leads to a new method of breaking  $\Theta$ . Unitary time evolution is an implicit assumption in the proof of the *CPT* theorem; Wald argued that non-unitary evolution would allow  $\Theta$  not to be a symmetry. Within the context of scattering theory let  $\mathcal{H}_{in}$  denote the Hilbert space of in states, and  $\bar{\mathcal{H}}_{in}$  the dual space. We can define the analogous entities for the out states. Let  $S$  be a mapping from the set of in-states  $\mathcal{G}_{in}$  to the set of out-states  $\mathcal{G}_{out}$ . In our framework  $\mathcal{G}_{in}$  is isomorphic to  $\mathcal{H} \otimes \bar{\mathcal{H}}_{in}$  since the states are represented as density matrices  $\rho_B^A$  where  $A$  is a vector index associated with  $\mathcal{H}_{in}$  and  $B$  is a vector index associated with  $\bar{\mathcal{H}}_{in}$ . The indexed form of  $S$  is  $S_{bC}^{aD}$  where the lower case indices refer to  $\mathcal{H}_{out}$  and  $\bar{\mathcal{H}}_{out}$ . If probability is conserved

$$\text{tr}(S\rho) = (\rho) \quad (1.12)$$

which in index notation can be written as  $S_{aC}^{aD} = \delta_C^D$  where we have adopted the summation convention for repeated indices. Consider now operators  $\Theta_{in}$  and  $\Theta_{out}$  which implement the *CPT* transformation on  $\mathcal{G}_{in}$  and  $\mathcal{G}_{out}$  respectively i.e.

$$\Theta_{in} : \mathcal{G}_{in} \rightarrow \mathcal{G}_{out} \quad (1.13)$$

$$\Theta_{out} : \mathcal{G}_{out} \rightarrow \mathcal{G}_{in}. \quad (1.14)$$

In particular under a *CPT* transformation let  $\rho_{in} \in \mathcal{G}_{in}$  be mapped into  $\rho'_{out}$  and  $\rho_{out} \in \mathcal{G}_{out}$  be mapped into  $\rho'_{in}$ . If the theory (including quantum gravity) is assumed to be invariant under *CPT* then

$$\rho_{out} = S\rho_{in}, \quad (1.15)$$

$$\rho'_{out} = S\rho'_{in} \quad (1.16)$$

$$\Theta_{in}\rho_{in} = \rho'_{out}, \quad (1.17)$$

$$\Theta_{out}\rho_{out} = \rho'_{in}, \quad (1.18)$$

and

$$\Theta_{in}\Theta_{out} = I, \quad \Theta_{out}\Theta_{in} = I. \quad (1.19)$$

Since

$$\Theta_{out} = \Theta_{in}^{-1} \quad (1.20)$$

it is convenient to drop the suffix *in* in this last relation. Hence from (1.15), (1.16), (1.17) and (1.18) we can deduce that

$$\Theta\rho_{in} = \rho'_{out}, \quad (1.21)$$

$$= S\rho'_{in} \quad (1.22)$$

$$= S\Theta^{-1}\rho_{out} \quad (1.23)$$

$$= S\Theta^{-1}S\rho_{in}, \quad (1.24)$$

From (1.21) and (1.24) we can deduce the important result that  $S$  has an inverse given by  $\Theta^{-1}S\Theta^{-1}$ . Hence if  $\Theta$  exists then time reversed evolution is permitted. Consequently for non-unitary evolution  $\Theta$  cannot be defined.

Another possible contribution of breakdown of *CPT* invariance is towards the generation of cosmological charge asymmetry (baryogenesis). This would provide an additional mechanism to that proposed by Sakharov based on: non-conservation of baryon number; deviation from thermal equilibrium; and *C* and *CP* violation. This has, in the past, been addressed within thermal equilibrium by using a mass difference between particle and anti-particle, a possible consequence of *CPT* violation. In our case of D-foam the decoherence effects on particle and anti-particle may be different, a fact we exploit.

## II. EFFECTIVE MODEL OF D-FOAM

For an observer on the brane world the crossing D-particles will appear as twinkling space-time defects, i.e. microscopic space-time fluctuations. This will give the four-dimensional brane world a ‘‘D-foamy’’ structure. In phase space,

for a D3-brane world, the function  $u_i$ , involving a momentum transfer,  $\Delta k_i$ , can be modelled by a local operator using the following parametrization [16]:

$$u_i = g_s \frac{\Delta k_i}{2M_s} = r_i k_i \text{ , no sum } i = 1, 2, 3 \text{ ,} \quad (2.1)$$

where the (dimensionful) variables  $r_i, i = 1, 2, 3$ , appearing above, are related to the fraction of momentum that is transferred at a collision with a D-particle in each spatial direction  $i$ . The target space-time metric state, which is close to being flat, can be represented schematically as a density matrix

$$\rho_{\text{grav}} = \int d^4 \tilde{r} \, f(\tilde{r}_\mu) |g(\tilde{r}_\mu)\rangle \langle g(\tilde{r}_\mu)|. \quad (2.2)$$

The parameters  $\tilde{r}_\mu$  ( $\mu = 0, \dots, 4$ ) are stochastic with a gaussian distribution  $f(\tilde{r}_\mu)$  characterised by the averages%

$$\langle \tilde{r}_\mu \rangle = 0, \quad \langle \tilde{r}_\mu \tilde{r}_\nu \rangle = \Delta_\mu \delta_{\mu\nu}. \quad (2.3)$$

The fluctuations experienced by the two entangled neutral mesons will be assumed to be independent and  $\Delta_\mu \sim O\left(\frac{E^2}{M_P^2}\right)$  i.e. very small. As matter moves through the space-time foam, assuming ergodicity, the effect of time averaging is assumed to be equivalent to an ensemble average. As far as our present discussion is concerned we will consider a semi-classical picture for the metric and so  $|g(\tilde{r}_\mu)\rangle$  in (2.2) will be a coherent state. In order to address two “flavours” the fluctuations of each component of the metric tensor  $g^{\alpha\beta}$  will not be simply given by the simple recoil distortion (1.4), but instead will be taken to have a  $2 \times 2$  (“flavour”) structure:

$$g_{00} = -1, \quad (2.4)$$

$$g_{01} = g_{10} = \tilde{r}_0 \sigma_0 + \tilde{r}_1 \sigma_1 + \tilde{r}_2 \sigma_2 \quad (2.5)$$

$$g_{jj} = 1 \quad (2.5)$$

where  $\sigma_0 = 1_2$ , (the identity matrix). The modelling of the metric can be made more elaborate, but the salient feature is the structure of  $g^{01}$  (where for simplicity we have taken the momentum to be in the 1-direction). For the case of the omega effect this restriction is acceptable since the K-mesons are produced collinearly in a  $\phi$ -meson factory (in the centre of mass frame). A more general ansatz than (2.4) (which will reduce to this when momenta are collinear) has the form

$$g_{0j} = r_\mu \sigma_\mu \hat{k}_j$$

where  $\hat{k}_j$  is the momentum operator of a particle moving in this background; this is the Finsler nature of the metric. We use the term flavour in a general sense. For K mesons flavour would mean  $K_L$  or  $K_S$ . An elementary D-particle will typically affect neutral particles because it does not carry charge and a charge string attached to it would not have anywhere for the flux to flow. Elementary D-particles appear in some string theories such as IIa but in other more phenomenologically relevant string theories they do not. In such cases, effective D-particle behaviour can occur from D3 branes wrapped around 3-cycles. Consequently such D-particles could interact with charged strings and hence contribute to baryogenesis. However the treatment of D-particles, arising from compactifications, is technically more involved and the orders of magnitude will depend on the details of the compactification process. We nonetheless will point out how the D-foam formally can give rise to particle-antiparticle asymmetry.

For the neutral Kaon system, the case of interest,  $K_0 - \bar{K}_0$ , is produced by a  $\phi$ -meson at rest, i.e.  $K_0 - \bar{K}_0$  in their C.M. frame. The CP eigenstates (on choosing a suitable phase convention for the states  $|K_0\rangle$  and  $|\bar{K}_0\rangle$ ) are, in standard notation,  $|K_\pm\rangle$  with

$$|K_\pm\rangle = \frac{1}{\sqrt{2}} (|K_0\rangle \pm |\bar{K}_0\rangle).$$

The mass eigensates  $|K_S\rangle$  and  $|K_L\rangle$  are written in terms of  $|K_\pm\rangle$  as

$$|K_L\rangle = \frac{1}{\sqrt{1 + |\varepsilon_2|^2}} [|K_-\rangle + \varepsilon_2 |K_+\rangle]$$

and

$$|K_S\rangle = \frac{1}{\sqrt{1+|\varepsilon_1|^2}} [|K_+\rangle + \varepsilon_1 |K_+\rangle].$$

In terms of the mass eigenstates

$$|i\rangle = \mathcal{C} \left\{ \begin{aligned} & \left( |K_L(\vec{k})\rangle |K_S(-\vec{k})\rangle - |K_S(\vec{k})\rangle |K_L(-\vec{k})\rangle \right) + \\ & \omega \left( |K_S(\vec{k})\rangle |K_S(-\vec{k})\rangle - |K_L(\vec{k})\rangle |K_L(-\vec{k})\rangle \right) \end{aligned} \right\}$$

where  $\mathcal{C} = \frac{\sqrt{(1+|\varepsilon_1|^2)(1+|\varepsilon_2|^2)}}{\sqrt{2(1-\varepsilon_1\varepsilon_2)}}$  [15]. In the notation of two level systems (on suppressing the  $\vec{k}$  label) we write

$$|K_L\rangle = |\uparrow\rangle \quad (2.6)$$

$$|K_S\rangle = |\downarrow\rangle. \quad (2.7)$$

These will be our “flavours” and represent the two physical eigenstates, with masses  $m_1 \equiv m_L$ ,  $m_2 \equiv m_S$ , with

$$\Delta m = m_L - m_S \sim 3.48 \times 10^{-15} \text{ GeV}. \quad (2.8)$$

The Klein-Gordon equation for a spinless neutral meson field  $\Phi = \begin{pmatrix} \phi_1 \\ \phi_2 \end{pmatrix}$  with mass matrix  $m = \frac{1}{2}(m_1 + m_2)1 + \frac{1}{2}(m_1 - m_2)\sigma_3$  in a gravitational background is

$$(g^{\alpha\beta} D_\alpha D_\beta - m^2)\Phi = 0 \quad (2.9)$$

where  $D_\alpha$  is the covariant derivative. Since the Christoffel symbols vanish for  $a_i$  independent of space time the  $D_\alpha$  coincide with  $\partial_\alpha$ . Hence

$$(g^{00}\partial_0^2 + 2g^{01}\partial_0\partial_1 + g^{11}\partial_1^2)\Phi - m^2\Phi = 0. \quad (2.10)$$

It is useful at this stage to rewrite the state  $|i\rangle$  in terms of the mass eigenstates.

The unnormalised state  $|i\rangle$  will then be an example of an initial state

$$|\psi\rangle = |k, \uparrow\rangle^{(1)} | -k, \downarrow\rangle^{(2)} - |k, \downarrow\rangle^{(1)} | -k, \uparrow\rangle^{(2)} + |\Delta\rangle \quad (2.11)$$

with

$$|\Delta\rangle = \xi |k, \uparrow\rangle^{(1)} | -k, \uparrow\rangle^{(2)} + \xi' |k, \downarrow\rangle^{(1)} | -k, \downarrow\rangle^{(2)}$$

where  $|K_L(\vec{k})\rangle = |k, \uparrow\rangle$  and we have taken  $\vec{k}$  to have only a non-zero component  $k$  in the  $x$ -direction; superscripts label the two separated detectors of the collinear meson pair,  $\xi$  and  $\xi'$  are complex constants and we have left the state  $|\psi\rangle$  unnormalised. The evolution of this state is governed by a hamiltonian  $\hat{H}$

$$\hat{H} = g^{01}(g^{00})^{-1}\hat{k} - (g^{00})^{-1}\sqrt{(g^{01})^2 k^2 - g^{00}(g^{11}k^2 + m^2)} \quad (2.12)$$

which is the natural generalisation of the standard Klein-Gordon hamiltonian in a one particle situation. Moreover  $\hat{k}|\pm k, \uparrow\rangle = \pm k|k, \uparrow\rangle$  together with the corresponding relation for  $\downarrow$ . We next note that the Hamiltonian interaction terms

$$\hat{H}_I = -(r_1\sigma_1 + r_2\sigma_2)\hat{k} \quad (2.13)$$

are the leading order contribution in the small parameters  $r_\mu$  in the Hamiltonian  $H$  (2.12), since the corresponding variances  $\sqrt{\Delta_\mu}$  are small. The term (2.13), has been used in [15] as a perturbation in the framework of non-degenerate perturbation theory, in order to derive the “gravitationally-dressed” initial entangled meson states, immediately after the  $\phi$  decay. The result is:

$$|k, \uparrow\rangle_{QG}^{(1)} | -k, \downarrow\rangle_{QG}^{(2)} - |k, \downarrow\rangle_{QG}^{(1)} | -k, \uparrow\rangle_{QG}^{(2)} = |\Sigma\rangle + |\tilde{\Delta}\rangle \quad (2.14)$$

where

$$|\Sigma\rangle = |k, \uparrow\rangle^{(1)} | -k, \downarrow\rangle^{(2)} - |k, \downarrow\rangle^{(1)} | -k, \uparrow\rangle^{(2)},$$

$$|\tilde{\Delta}\rangle = |k, \downarrow\rangle^{(1)} | -k, \downarrow\rangle^{(2)} (\beta^{(1)} - \beta^{(2)}) + |k, \uparrow\rangle^{(1)} | -k, \uparrow\rangle^{(2)} (\alpha^{(2)} - \alpha^{(1)}) \\ + \beta^{(1)} \alpha^{(2)} |k, \downarrow\rangle^{(1)} | -k, \uparrow\rangle^{(2)} - \alpha^{(1)} \beta^{(2)} |k, \uparrow\rangle^{(1)} | -k, \downarrow\rangle^{(2)}$$

and

$$\alpha^{(i)} = \frac{\langle \uparrow, k^{(i)} | \widehat{H}_I | k^{(i)}, \downarrow \rangle^{(i)}}{E_2 - E_1}, \quad \beta^{(i)} = \frac{\langle \downarrow, k^{(i)} | \widehat{H}_I | k^{(i)}, \uparrow \rangle^{(i)}}{E_1 - E_2}, \quad i = 1, 2 \quad (2.15)$$

where the index  $(i)$  runs over meson species (“flavours”) ( $1 \rightarrow K_L$ ,  $2 \rightarrow K_S$ ). The reader should notice that the terms proportional to  $(\alpha^{(2)} - \alpha^{(1)})$  and  $(\beta^{(1)} - \beta^{(2)})$  in (\ref{entangled}) generate  $\omega$ -like effects. We concentrate here for brevity and concreteness in the strangeness conserving case of the  $\omega$ -effect in the initial decay of the  $\phi$  meson [15], which corresponds to  $r_i \propto \delta_{i2}$ . We should mention, however, that in general quantum gravity does not have to conserve this quantum number, and in fact strangeness-violating  $\omega$ -like terms are generated in this problem through time evolution [15].

We next remark that, on averaging the density matrix over the random variables  $r_i$ , which are treated as independent variables between the two meson particles of the initial state, we observe that only terms of order  $|\omega|^2$  will survive, with the order of  $|\omega|^2$  being

$$|\omega|^2 = \tilde{\Delta}_{(1),(2)} \left( \mathcal{O} \left( \frac{1}{(E_1 - E_2)} (\langle \downarrow, k | H_I | k, \uparrow \rangle)^2 \right) \right), \\ = \tilde{\Delta}_{(1),(2)} \left( \mathcal{O} \left( \frac{\Delta_2 k^2}{(E_1 - E_2)^2} \right) \right) \sim \tilde{\Delta}_{(1),(2)} \left( \frac{\Delta_2 k^2}{(m_1 - m_2)^2} \right) \quad (2.16)$$

for the physically interesting case of non-relativistic Kaons in  $\phi$  factories, in which the momenta are of order of the rest energies. The notation  $\tilde{\Delta}_{(1),(2)}(\dots)$  above indicates that one considers the differences of the variances  $\Delta_2$  between the two mesons 1 - 2, in that order.

The variances in our model of D-foam, which are due to quantum fluctuations of the recoil velocity variables about the zero average (dictated by the imposed requirement on Lorentz invariance of the string vacuum) lead for the square of the amplitude of the (complex)  $\omega$ -parameter:

$$|\omega|^2 \sim g_0^2 \frac{(m_1^2 - m_2^2)}{M_s^2} \frac{k^2}{(m_1 - m_2)^2} = \frac{m_1 + m_2}{m_1 - m_2} \frac{k^2}{(M_s^2/g_0^2)}, \quad (2.17)$$

where  $M_P \equiv M_s/g_0$  represents the (average) quantum gravity scale, which may be taken to be the four-dimensional Planck scale. In general,  $M_s/g_0$  is the (average) D-particle mass, as already mentioned. In the modern version of string theory,  $M_s$  is arbitrary and can be as low as a few TeV, but in order to have phenomenologically correct string models with large extra dimensions one also has to have in such cases very weak string couplings  $g_0$ , such that even in such cases of low  $M_s$ , the D-particle mass  $M_s/g_0$  is always close to the Planck scale  $10^{19}$  GeV. But of course one has to keep an open mind about ways out of this pattern, especially in view of the string landscape.

The result (2.17), implies, for neutral Kaons in a  $\phi$  factory, for which (\ref{deltam}) is valid, a value of:  $|\omega| \sim 10^{-11}$ , which in the sensitive  $\eta^{+-}$  bi-pion decay channel, is enhanced by three orders of magnitude, as a result of the fact that the  $|\omega|$  effect always appears in the corresponding observables [15] in the form  $|\omega|/|\eta^{+-}|$ , and the CP-violating



parameter  $|\eta^{+-}| \sim 10^{-3}$ . Unfortunately, this value is still some two orders of magnitude away from current bounds of the  $\omega$ -effect at, or the projected sensitivity of upgrades of, the DAΦNE detector.

A similar calculation can be done for the particle-antiparticle asymmetry except that we will use the more general metric resulting in

$$H = E_r \left( \vec{k} \right) 1_2 + r_\mu \sigma_\mu \left( \vec{k} \right)^2 \quad (2.18)$$

where

$$E_r \left( \vec{k} \right) = \sqrt{m^2 + \sum_j k_j^2 + (r_\mu)^2 \left( \sum_j k_j^2 \right)^2 - \sum_j r_j^2 k_j^4}$$

This leads to gravitational dressing of the particle and antiparticle states so that the masses get shifted so as to induce a mass difference; in thermodynamic equilibrium the canonical number distribution ( Bose-Einstein or Fermi-Direc) would give a difference between the particle and anti-particle number. Explicitly, if the eigenvalues of  $H$  are denoted by  $a_1 \left( \vec{k}, r_\mu \right)$  and  $a_2 \left( \vec{k}, r_\mu \right)$  then to lowest order we can show that  $a_2 \left( \vec{k}, r_\mu \right) - a_1 \left( \vec{k}, r_\mu \right) = 2 \left| \vec{k} \right| \left| \vec{r} \right|$  and clearly there will then be a difference between the grand canonical number distribution functions

$$n_{a_j} \left( \vec{k}, r_\mu, \xi \right) = \frac{1}{\exp \left( \beta \left[ a_j \left( \vec{k}, r_\mu \right) - \mu \right] \right) + \xi}, \quad j = 1, 2$$

where  $\xi = \pm 1$  for different  $j$ .

## Conclusions

In our D-foam model, string matter on a brane world interacts with D-particles in the bulk. Recoil of the heavy D particles owing to interactions with the stringy matter produces a gravitational distortion which has a back-reaction on matter. There is information loss at the recoil which leads to non-unitary evolution for the matter. From the general considerations of Wald, CPT symmetry may then be violated. This new mechanism of violation is explored for the omega effect as well as matter-anti-matter asymmetry. For the omega effect there is a chance that the upgrade to the the DAΦNE detector will allow a stringent test of the D-foam prediction. Particle-antiparticle asymmetry has more theoretical uncertainties in D-foam predictions and further investigations are needed using D-brane instantons for example.

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## Appendix A: Calculation of back reaction in D-particle foam

### D-particle foam contributions to master equation for Liouville-decoherence

The material in this Appendix is a review based on [4], where we refer the reader for further details. Let us consider a  $D$ -particle, located at  $y^i(t=0) \equiv y_i$  of the spatial coordinates of a  $(d+1)$ -dimensional space time (which could be a D3-brane world), which at a time  $t=0$  experiences an impulse, as a result of scattering with a matter string state . In a  $\sigma$ -model framework, the trajectory of the  $D$ -particle  $y^i(t)$ ,  $i = 1, 2, \dots, d$ , a spatial index, is described by inserting the following vertex operator in the  $\sigma$ -model of a free string:

$$V = \int_{\partial\Sigma} g_{ij} y^j(t) \partial_n X^i \quad (2.19)$$

where  $g_{ij}$  denotes the spatial components of the metric,  $\partial\Sigma$  denotes the world-sheet boundary,  $\partial_n$  is a normal world-sheet derivative,  $X^i$  are  $\sigma$ -model fields obeying Dirichlet boundary conditions on the world sheet, and  $t$  is a  $\sigma$ -model field obeying Neumann boundary conditions on the world sheet, whose zero mode is the target time. The space-time prior to Liouville dressing is assumed Euclidean for formal reasons (convergence of the corresponding  $\sigma$ -model path integral). We note, however, that the final Liouville-dressed target space-time acquires Minkowski signature as a result of the time-like signature of the Liouville mode [17].

In the non-relativistic approximation, appropriate for a heavy D-particle defect of mass  $M_s/g_s$ , with  $M_s$  the string scale, and  $g_s$  the string coupling, assumed weak ( $g_s \ll 1$ ), the path  $y^i(t)$  corresponding to the impulse is given by:

$$y_i(t) = (\varepsilon y_i + u_i t) \Theta_\varepsilon(t) \quad (2.20)$$

$$u_i = (k_1 - k_2)_i, \quad (2.21)$$

with  $k_1(k_2)$  the momentum of the propagating string state before (after) the recoil  $y_i$  are the spatial collective coordinates of the D particle, and the regularized Heaviside functional operator  $\Theta_\varepsilon(t)$  is given by (1.2) in the text [4]:

$$\Theta_\varepsilon(t) = \frac{1}{2\pi i} \int_{-\infty}^{\infty} \frac{dq}{q - i\varepsilon} e^{iqt}, \quad (2.22)$$

Eq. (2.20) contains actually a *pair* of deformations corresponding to the  $\sigma$ -model couplings  $y_i$  and  $u_i$ . These deformations are relevant in a world-sheet renormalization-group sense, having anomalous scaling dimension  $-\frac{\varepsilon^2}{2}$ , i.e. to leading order in a coupling constant expansion their renormalization-group  $\beta$ -functions read:

$$\beta^{y^i} = -\frac{\varepsilon^2}{2} y^i, \quad \beta^{u^i} = -\frac{\varepsilon^2}{2} u^i. \quad (2.23)$$

The deformations form a logarithmic conformal algebra (superconformal algebra in the case of superstrings) which *closes* if and only if one identifies [4] the regulating parameter  $\varepsilon^{-2}$  with the world-sheet renormalization-group scale  $\ln|L/a|^2$  ( $L(a)$  is the Infrared (Ultraviolet) world-sheet scale):

$$\varepsilon^{-2} = \eta \ln|L/a|^2 \quad (2.24)$$

where  $\eta$  denotes the signature of time  $t$  of the target-space manifold of the  $\sigma$ -model (prior to Liouville dressing). For Euclidean manifolds, assumed here for path-integral convergence,  $\eta = +1$ .

Upon the identification (2.24) the re-scaled couplings  $\bar{y}_i \equiv \frac{y_i}{\varepsilon}$  and  $\bar{u}_i \equiv \frac{u_i}{\varepsilon}$  are *marginal*, that is independent of the scale  $\varepsilon$ . It is these marginal couplings that are connected to target-space quantities of physical significance, such as the space-time back reaction of recoil. In the limit  $\varepsilon \rightarrow 0$  ( the long-time limit) the dominant contributions come from the  $u_i$  recoil deformation in (2.20).

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- [1] see, for instance: D. Bao, S. S. Chern and Z. Shen, *An introduction to Finsler Geometry* (Springer-Verlag (NY, 2000)); For a recent review, with applications to cosmology, see S. Vacaru, “Principles of Einstein-Finsler Gravity and Cosmology,” arXiv: 1012. 4148 [physics.gen-ph].
- [2] J. Polchinski, “Dirichlet branes and Ramond-Ramond charges”, Phys. Rev. Lett. 75 4724 (1995).
- [3] J. R. Ellis, N. E. Mavromatos and M. Westmuckett, “A supersymmetric D-brane model of space-time foam,” Phys. Rev. D 70, 044036 (2004) [arXiv:gr-qc/0405066]; “Potentials between D-branes in a supersymmetric model of space-time foam”, Phys. Rev. D 71, 106006 (2005) [arXiv:gr-qc/0501060].
- [4] I. I. Kogan, N. E. Mavromatos and J. F. Wheeler, “D-brane recoil and logarithmic operators,” Physics Letters B 387, 483 (1996) [arXiv:hep-th/9606102]; For a review focusing on D-brane recoil, including supermembranes, see: N. E. Mavromatos, “Logarithmic conformal field theories and strings in changing backgrounds,” arXiv:hep-th/0407026, in Shifman, M. (ed.) et al.: From fields to strings, I. Kogan memorial Volume 2}, 1257-1364. (World Sci. 2005), and references therein.
- [5] L. A. Anchordoqui, “Spacetime foam at a TeV,” J. Phys. Conf. Ser. 60 (2007) 191 [arXiv:hep-ph/0610025].
- [6] N. Arkani-Hamed, S. Dimopoulos and G. R. Dvali, “The hierarchy problem and new dimensions at a millimeter,” Phys. Lett. B 429, 263 (1998) [arXiv:hep-ph/9803315].
- [7] J. R. Ellis, N. E. Mavromatos and D. V. Nanopoulos, Gen. Rel. Grav. **32**, 127 (2000) [arXiv:gr-qc/9904068]; Phys. Rev. D **61**, 027503 (2000) [arXiv:gr-qc/9906029]; Phys. Rev. D **62**, 084019 (2000) [arXiv:gr-qc/0006004]. J. R. Ellis, K. Farakos, N. E. Mavromatos, V. A. Mitsou and D. V. Nanopoulos, Astrophys. J. **535**, 139 (2000) [arXiv:astro-ph/9907340]; J. R. Ellis, N. E. Mavromatos, D. V. Nanopoulos and G. Volkov, Gen. Rel. Grav. **32**, 1777 (2000) [arXiv:gr-qc/9911055]. J. R. Ellis, N. E. Mavromatos and M. Westmuckett, Phys. Rev. D **70**, 044036 (2004) [arXiv:gr-qc/0405066].
- [8] G. M. Shore, “A local renormalization group equation, diffeomorphisms and conformal invariance in sigma-models”, Nuclear Physics B **286** 349 (1987)

- [9] A. B. Zamolodchikov, “‘Irreversibility’ Of The Flux Of The Renormalization Group In A 2-D Field Theory,” JETP Lett. 43, 730 (1986) [Pisma Zh. Eksp. Teor. Fiz.43, 565 (1986)].
- [10] M. Gutperle, M. Headrick, S. Minwalla and M. Schomerus, “Space-time energy decreases under world-sheet renormalization group flow”, Journal of High Energy Physics, 1 073 (2003)
- [11] J. Ellis, N. E. Mavromatos and D. V. Nanopoulos, “A non-critical string approach to black holes, time and quantum dynamics”, in “From supersymmetry to the origin of space-time” Ed A. Zichichi, World Scientific (1995), arXiv:hep-th/9403133
- [12] N. E. Mavromatos, “Decoherence and CPT Violation in a Stringy Model of Space-Time Foam,” Found.Phys. 40, 917 (2010) [arXiv:0906.2712 [hep-th]].
- [13] R. F. Streater and A. S. Wightman, PCT, Spin and Statistics, and All That, Benjamin, New York, (1964); O. W. Greenberg, “Why is CPT fundamental?,” Found. Phys. 36, 1535 (2006) [arXiv:hep-ph/0309309].
- [14] H. J. Lipkin, “CP violation and coherent decays of kaon pairs,” Phys. Rev. 176 1715 (1968)
- [15] J. Bernabeu, N. E. Mavromatos and S. Sarkar, “Decoherence induced CPT violation and entangled neutral mesons,” Physical Review D 74, 045014 (2006) [arXiv:hep-th/0606137].
- [16] N.E. Mavromatos and Sarben Sarkar, “Liouville decoherence in a model of flavour oscillations in the presence of dark energy,” Physical Review D 72, 065016 (2005) [arXiv:hep-th/0506242].
- [17] E. Gravanis and N. E. Mavromatos, “Vacuum energy and cosmological supersymmetry breaking in brane worlds,” Physics Letters B 547, 117 (2002) [arXiv:hep-th/0205298].